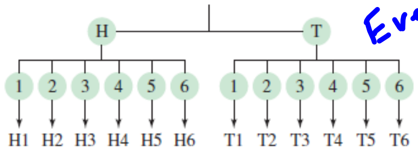
Example 1:

A probability experiment consists of tossing a coin and rolling a six-sided die.

a. Draw a tree diagram to represent the sample space.

Tree Diagram for Coin and Die Experiment



Event 2

Event 6

b. How many outcomes are in the sample space?

12

c. List the sample space.

From the tree diagram, the sample space has 12 outcomes.

{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

{H1, H2, . . . , H6, T1, T2, . . . T6}

$2 \cdot 6 = 12$

A **simple event** is an event that _ An event that consists of a single outcome

Try It Yourself 2:

You ask for a student's age at his or her last birthday. Listed below are two different events:

Event C: The student's age is between 18 and 23, inclusive 6 outcomes Not simple

Event D: The student's age is 20. simple event

For each event, decide how many outcomes are in the event and whether the event is *simple* or not.

The Fundamental Counting Principal

- Independent Events , then multiply each event's sample spaces together

Example

Your choices now include a ~~Toyota, a van, or a tan or gray~~ car. Now, how many different ways can you select a car?

van, truck, car — yellow, red

$3 \cdot 2 = 6$

Example 4:

The PIN (personal identification number) for an ATM withdrawal consists of four digits. Each digit can be 0 through 9.

0-9 ⇒ 10

How many access codes are possible if

a. each digit can be used only once? $10 \cdot 9 \cdot 8 \cdot 7$

b. each digit can be repeated? $10 \cdot 10 \cdot 10 \cdot 10 = 10,000$

c. each digit can be repeated but thei first digit cannot be 0 or 1? $8 \cdot 10 \cdot 10 \cdot 10 = 8,000$

DEFINITION

Classical (or **theoretical**) **probability** is used when each outcome in a sample space is equally likely to occur. The classical probability for an event E is given by

$$P(E) = \frac{\text{Number of outcomes in event } E}{\text{Total number of outcomes in sample space}}.$$

Example 5:

You roll a six-sided die. What is the sample space? {1, 2, 3, 4, 5, 6}

Find the probability of the following events:

1. Event A: rolling a 5 1/6
2. Event B: rolling an 8 0
3. Event C: rolling a number greater than 1 5/6

complement
1 - P(1) = 1 - 1/6
= 5/6

Law of Large Numbers Click her for Notes

Empirical (or ___________) Probability is based on _____________________ from probability experiments. List the formula for finding empirical probability.

Empirical (or **statistical**) **probability** is based on observations obtained from probability experiments. The empirical probability of an event E is the relative frequency of event E.

$$P(E) = \frac{\text{Frequency of event } E}{\text{Total frequency}} = \frac{f}{n}.$$

Example 6:

A company is conducting an online survey of randomly selected individuals to determine if traffic congestion is a problem in their community. The results are listed below. What is the probability that the next person that responds to the survey says that traffic congestions is a serious problem in their community?

Response	Number of times
It is not a problem.	82
It is a moderate problem.	115
It is a serious problem	123

Response	Number of times, f
It is a serious problem.	123
It is a moderate problem.	115
It is not a problem.	82
	$\Sigma f = 320$

$\Sigma f =$ 82 + 115 + 123 = 320

sample space
total outcomes

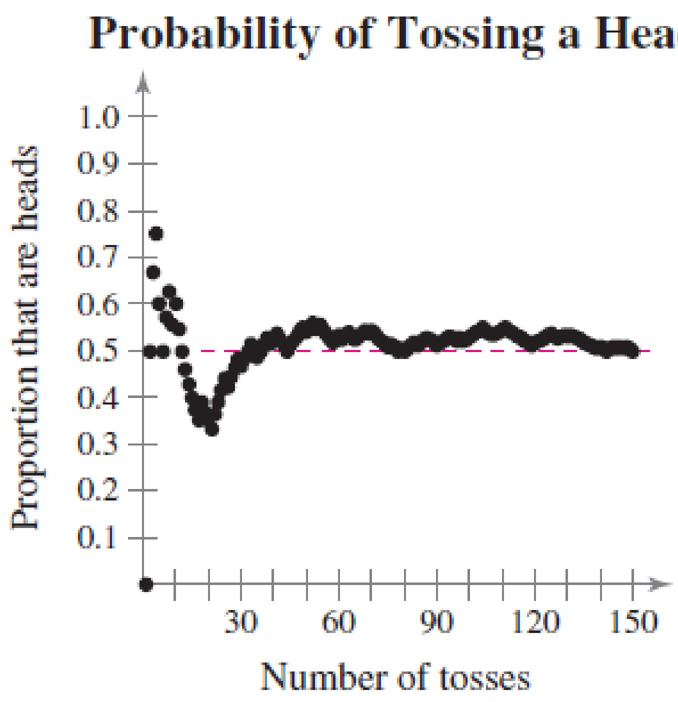
123/320 = 38%
.38

Die H T = 2
Coin 1 2 3 4 5 6 1 2 3 4 5 6 =
Card A K A K

As you increase the number of times a probability experiment is repeated, the empirical probability (relative frequency) of an event approaches the theoretical probability of the event. This is known as the **law of large numbers.**

LAW OF LARGE NUMBERS

As an experiment is repeated over and over, the empirical probability of an event approaches the theoretical (actual) probability of the event.



Summary of probability:

A probability cannot be negative or greater than 1

So the probability of an event E is between 0 and 1 or 0 and 100 %

That is, $0 \le P(E) \le 1$.

Complement of Event E

You survey a sample of 1000 employees at a company and record the age of each. The results are shown in the frequency distribution. If you randomly select an employee, what is the probability they are not between 25 and 34 years old?

Employees Age	15 to 24	25 to 34	35 to 44	45 to 54	55 to 64	65 and over
Frequency	54	366	233	180	125	42

$\Sigma f = 1{,}000$ $\frac{634}{1{,}000} = \boxed{\frac{317}{500}}$

P(not between 25-34)

1 - P(between 25-34) = $1 - \frac{366}{1{,}000}$

Conditional Probability

"given"

This is denoted by P(B|A) and is read as probability of B given A.

Example 1: Use the table below to find the following probabilities:

	Rides bus	Drives	Other	
Freshmen	25	0	17	42
Sophomores	19	2	12	33
Juniors	15	7	11	33
Seniors	12	13	9	34
	71	22	49	142

a. What is the probability of choosing a freshman? 42/142

b. What is the probability of choosing a person that drives to school? 22/142

c. What is the probability that a student drives to school, given they are a junior? $P(D|J) = \frac{D \,\&\, J}{J} = \frac{7}{33}$

d. What is the probability that a student is a freshmen, given they ride the bus to school?

e. P(senior)

f. P(other|freshmen) $= \frac{17}{42}$

g. P(sophomore|drive) $\frac{2}{22}$

$P(F|B) = \frac{F \,\&\, B}{B}$ $\boxed{= \frac{25}{71}}$

$(F|B)$

Given they ride the bus to school, what is the probability the student is a freshman.

Independent Events F

Family C

> **DEFINITION**
>
> Two events are **independent** if the occurrence of one of the events does not affect the probability of the occurrence of the other event. Two events A and B are independent if
>
> $$P(B|A) = P(B) \quad \text{or if} \quad P(A|B) = P(A).$$
>
> Events that are not independent are **dependent.**

Events that are not independent are **dependent.**

J K R J

Example 2:

Decide whether the events are independent or dependent.

a. Practicing the piano (A) and then becoming a concert pianist (B). D

b. Tossing a coin and getting a tail (A), and then rolling a six-sided die and obtaining a 3 (B). I

c. A salmon swims successfully through a dam (A) & then swims successfully through a second dam (B). D

d. Exercising frequently (A) and having a low resting heart rate (B). D

e. Driving over 85 miles per hour (A), and then getting in a car accident (B). D

$P(H \text{ and } 3)$

$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

III. The Multiplication Rule for the Probability of A and B Definition

If events A and B are dependent, then the rule for multiplication is $P(A) \cdot P(B|A)$

If events A and B are independent, then the rule for multiplication is $P(A) \cdot P(B)$

and ⇒ multiply

A B

AND

Example 3

a. Two cards are selected without replacement from a standard deck of cards. Find the probability of selecting a king and then selecting a queen.

$\frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} = \frac{4}{663}$

b. A coin is tossed and a die is rolled. Find the probability of getting a tail and then rolling a 3.

$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

c. Suppose the probability of a salmon successfully swimming through a dam is 0.90.

1. Find the probability that three salmon swim successfully through the dam.

$(.9) \cdot (.9) \cdot (.9) = .729$

2. Find the probability that at least one salmon swims successfully through the dam.

$1 - P(\text{none})$

d. Two cards are selected without replacement from a standard deck of cards. Find the probability that they are both hearts.

$P(B|A)$

$P(H \text{ and } H)$

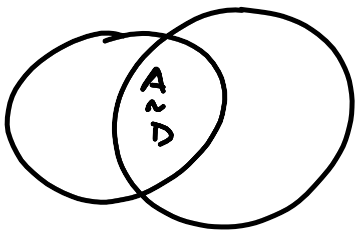
$\frac{26}{52} \cdot \frac{25}{51}$

THE MULTIPLICATION RULE FOR THE PROBABILITY OF A AND B

The probability that two events A and B will occur in sequence is

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A).$$

If events A and B are independent, then the rule can be simplified to $P(A \text{ and } B) = P(A) \cdot P(B)$. This simplified rule can be extended for any number of independent events.



Multiplication Rule click

1. Two events A and B are **mutually exclusive** if A and B cannot occur at the same time.

Example 1: Decide if the events are mutually exclusive.

a. Select a card from a standard deck A: The card is a jack B: The card is a face card

b. Select a student: A: The student is 20 years old. B. The student has blue eyes.

c. Selected a registered vehicle: A: The vehicle is a Ford. B. The vehicle is a Toyota.

2. **The Addition Rule for the Probability of A or B** click

If events A and B are mutually exclusive, then the rule for addition is P(A) + P(B).

If events A and B are not mutually exclusive, then the rule for addition is P(A) + P(B) - P(A and B)

Example 2:

a. You select a card from a standard deck. Find the probability that the card is a 9 or a King.

b. You roll a die. Find the probability of rolling a number greater than 3 or an odd number.

c. A card is selected from a standard deck. Find the probability that the card is a 10 or a heart.

Example 3: A blood bank catalogs the types of blood, including positive or negative Rh-factor, given by donors during the last five days. The number of donors who gave each blood type is shown in the table.

		O	A	B	AB
RH – factor	**Positive**	156	139	37	12
	Negative	28	25	8	4

A donor is selected at random.

1. Find the probability that the donor has type O or type A blood.
2. Find the probability that the donor has type B blood or is Rh-negative.
3. Find the probability that the donor has type B or type AB blood.
4. Find the probability that the donor has type A blood or is Rh-positive.

THE ADDITION RULE FOR THE PROBABILITY OF *A* OR *B*

The probability that events A or B will occur, $P(A \text{ or } B)$, is given by

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

If events A and B are mutually exclusive, then the rule can be simplified to $P(A \text{ or } B) = P(A) + P(B)$. This simplified rule can be extended to any number of mutually exclusive events.

THE MULTIPLICATION RULE FOR THE PROBABILITY OF A AND B

The probability that two events A and B will occur in sequence is

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A).$$

If events A and B are independent, then the rule can be simplified to $P(A \text{ and } B) = P(A) \cdot P(B)$. This simplified rule can be extended for any number of independent events.

Permutations

The number of permutations of n distinct objects taken r at a time is

$${}_nP_r = \frac{n!}{(n-r)!}, \text{ where } r \leq n.$$

Example 1:
Find the number of ways of forming three-digit codes in which no digit is repeated.

Example 2:
Forty-three race cars started the 2007 Daytona 500. How many ways can the cars finish first, second, and third?

Example 3:
The board of directors for a company has twelve members. You are going to elect a president, vice-president, secretary and treasurer. How many ways can these positions be assigned?

Combinations

COMBINATION OF n OBJECTS TAKEN r AT A TIME

A combination is a selection of r objects from a group of n objects without regard to order and is denoted by ${}_nC_r$. The number of combinations of r objects selected from a group of n objects is

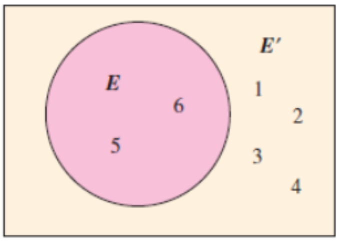
$${}_nC_r = \frac{n!}{(n-r)!r!}.$$

Example 1:
A state department of transportation plans to develop a new section of interstate highway and receives 16 bids for the project. The state plans to hire four of the bidding companies. How many different combinations of four companies can be selected from the 16 bidding companies?

Example 2: The manager of an accounting department wants to form a three-person advisory committee from the 20 employees in the department. In how many ways can the manager do this?

Complementary Events

The sum of the probabilities of all outcomes in a sample space is 1 or 100%. An important result of this fact is that if you know the probability of an event E, you can find the probability of the *complement of event* E.



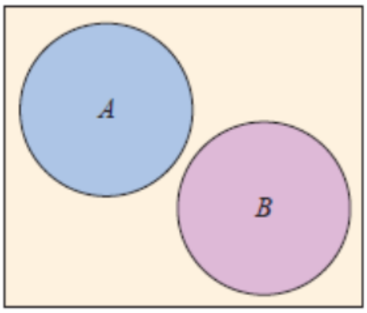
DEFINITION

The **complement of event** E is the set of all outcomes in a sample space that are not included in event E. The complement of event E is denoted by E' and is read as "E prime."

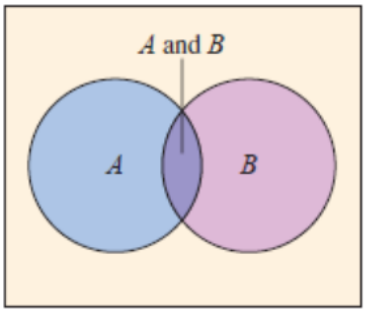
DEFINITION

Two events A and B are **mutually exclusive** if A and B cannot occur at the same time.

The Venn diagrams show the relationship between events that are mutually exclusive and events that are not mutually exclusive.



A and B are mutually exclusive.



A and B are not mutually exclusive.

Type of Probability and Probability Rules	In Words	In Symbols
Classical Probability	The number of outcomes in the sample space is known and each outcome is equally likely to occur.	$P(E) = \frac{\text{Number of outcomes in event } E}{\text{Number of outcomes in sample space}}$
Empirical Probability	The frequency of outcomes in the sample space is estimated from experimentation.	$P(E) = \frac{\text{Frequency of event } E}{\text{Total frequency}} = \frac{f}{n}$
Range of Probabilities Rule	The probability of an event is between 0 and 1, inclusive.	$0 \le P(E) \le 1$
Complementary Events	The complement of event E is the set of all outcomes in a sample space that are not included in E, denoted by E'.	$P(E') = 1 - P(E)$
Multiplication Rule	The Multiplication Rule is used to find the probability of two events occurring in a sequence.	$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ *Independent events*
Addition Rule	The Addition Rule is used to find the probability of at least one of two events occurring.	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = P(A) + P(B)$ *Mutually exclusive events*