Geometric Distribution

Definition:

Geometric distribution is a probability model and statistical data that is used to find out the number of failures which occurs before single success

Formula:

$P(x) = q^x p$

q = failure (1 − p)

p = success

Where ,

p = probability of success for a single trial

q = probability of failure for a single trial (= 1-p)

x = number of failures (= 0, 1, 2 ...)

p+q = 1

The geometric distribution is a discrete distribution for $n = 0, 1, 2, \ldots$ having probability density function

$$P(n) = p(1-p)^n = p\,q^n,$$

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Remember:

p = probability of success

q = probability of failure

$P(X = 1) = p$ No failure,success on first attempt S

$P(X = 2) = q \times p$ 1 failure then success FS qp

$P(X = 3) = q^2 \times p$ 2 failures then success FFS qqp

$P(X = 4) = q^3 \times p$ 3 failures then success FFFS $qqqp$ q^3p

$P(X = r) = q^{r-1} \times p$ r - 1 failures followed by success

$q^{x-1}p$

If X follows a Geometric distribution, we write:

X ~ Geo(p)

This reads as **'X has a geometric distribution with probability of success, p'.**

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Example 1 :
A boy rolling a die. Calculate the probability of getting 3 on the 6th roll.

Step 1: Let us first calculate p which is the probability of success for a single trial
$p = 1/6 = 0.166$.

Step 2: Now, find the value of q.
q = probability of failure for a single trial ($= 1-p$)
$q = (1 - 1/6)$
$q = 0.834$. (Note : $p + q = 1$)

Step 3: Calculate the value of x.
x denotes the number of failures before a success.
$x= 6-1 \Rightarrow x = 5$.

Step 4: Now, substitute the value of p, q and x in the formula,
$\mathbf{P(x) = q^x p}$
$P(5) = (0.834^4 * 0.166) = 0.066914$.

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Expectation and Variance

If $X \sim Geo(p)$, then:

- $E(X) = 1/p$
- $Var(X) = q/p^2$, where $q = 1 – p$

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Ex. 2

Find the probability of getting a success on the 7^{th} sales call given:

X	Y	Z
a) $p = 30\%$	b) $p = 10\%$	c) $p = 80\%$
$q = 70\%$	
$(.70)^6(.30)$	$(.90)^6(.10)$	$(.20)^6(.80)$
$= .035$	$.053$	$.0000512$

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X

binomial

$n = 5$

$p = 20\% \rightarrow .20$

$\mu = E(x) = 5(.20) = 1$

Y

$n = 7$

$p = 40\%$

$7(.40)$

$E(Y) = .28$

I. $E(x) = E(Y)$ ✓

II. $E(x) > E(Y)$

III. $E(Y) > E(x)$

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