

UNIT 4A REVIEW

RANDOMNESS AND PROBABILITY
CHAPTERS 14 & 15

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PROBABILITY RULES

- $0 \leq P(\text{any event}) \leq 1$
- $\sum P(\text{all events}) = 1$ (\sum is summation)
- $P(\text{certainty}) = 1$ and $P(\text{impossibility}) = 0$
- **Complement Rule:** $P(\text{not occurring}) = 1 - P(\text{occurring})$
- **Disjoint Events Rule:** If two events have no outcomes in common (disjoint), then $P(A \text{ or } B) = P(A) + P(B)$
- **Multiplication Rule:** If A and B are independent (knowing if one event occurs does not change the probability of the other event), then $P(A \text{ and } B) = P(A) \times P(B)$
- **At least Probabilities:**
 $P(\text{at least one}) = 1 - P(\text{complement of "at least one"})$
 $= 1 - P(\text{none})$

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GENERAL MULTIPLICATION RULE

General Multiplication Rule: Probability that two events A and B both occur (regardless of independence) is

$$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B | A)$$

where $P(B | A)$ is a conditional probability read as the probability of B **given that A has occurred**

Conditional Probabilities: If A and B are any two events, then

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{N(A \text{ and } B)}{N(A)}$$

where N is the number of outcomes

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INDEPENDENCE IN TERMS OF CONDITIONAL PROBABILITY

Two events A and B are independent if $P(B | A) = P(B)$

Example: $P(A = \text{Rolling a six on a single die}) = 1/6$

$P(B = \text{Rolling a six on a second roll}) = 1/6$
no matter what was rolled on the first roll!!!

So probability of rolling a 6 on the second roll, given you rolled a six on the first is still $1/6$

$P(B | A) = P(B)$ so A and B are independent

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TREE DIAGRAM



- Tree diagrams can be used to determine right-end (conditional) probabilities by multiplying the probabilities as you get to the right end
- Good analysis tool for AP problems

CONTINGENCY TABLES

	Male	Female	Total
Right handed	48	42	90
Left handed	12	8	20
Total	60	50	110

Contingency tables are easy ways to show conditional probabilities. Givens eliminate other rows and columns from the calculations as seen in the examples below:

- What is the probability of left-handed given that it is a male?
 $P(LH | M) = 12/60 = 0.20$
- What is the probability of female given that they were right-handed?
 $P(F | RH) = 42/90 = 0.467$
- What is the probability of being left-handed?
 $P(LH) = 20/110 = 0.182$

PROBLEM 1A

At a recent movie, 1000 patrons (560 females and 440 males) were asked whether or not they liked the film. It was determined that 355 females liked the film and 250 males said they enjoyed it also. If a person is randomly selected from the moviegoers what is the probability that the moviegoer was:

a) male?

$$P(M) = 440/1000 = 0.44$$

b) a female and liked the film

$$P(F \& L) = 355/1000$$

c) a male or someone who disliked the film?

$$440 + 205/1000$$

d) a male and disliked the film?

$$P(M \& DL) = 190/1000$$

e) a male given they liked the film?

$$P(M|L) = 250/605$$

f) someone who liked the film given they were a female?

$$P(L|F) = 355/560$$

g) Are sex and film preference independent?

Yes, if $P(M|L) = P(M)$, or other equivalent statement

	Liked	Disliked
Male	250	190
Female	355	205

PROBLEM 1B

Mutually Exclusive $\rightarrow P(A \text{ and } B) = 0!!$

If $P(A) = .3$ and $P(B) = .45$ and events A and B are mutually exclusive find $P(A \text{ or } B)$.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = 0.3 + 0.45 = 0.75$$

PROBLEM 1C

When spot-checked for safety, automobiles are found to defective tires 15% of the time, defective lights 25% of the time, and both defective tires and lights 8% of the time. Find the probability that a randomly chosen car has defective lights given that its tires are found to be defective.

$$P(DL \mid DT) = P(DL \text{ and } DT) / P(DT)$$

$$P(DL \mid DT) = 0.08 / 0.15 = 0.5333$$

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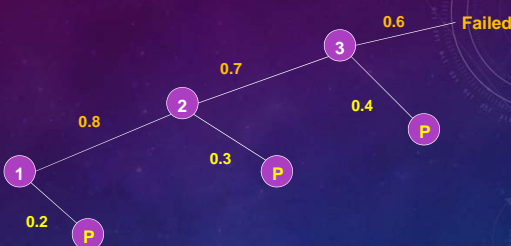
PROBLEM 2

Elaine is enrolled in a self-paced course that allows three attempts to pass an exam on the material. She does not study and has probability 0.2 of passing on the first try. If she fails on the first try, her probability of passing on the second try increases to 0.3 because she learned something on the first attempt. If she fails on two attempts, the probability of passing on a third attempt is 0.4. She will stop as soon as she passes. The course rules force her to stop after three attempts in any case.

Draw a tree diagram to illustrate what is described above, and use it to determine the probability that Elaine passes the exam.

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PROBLEM 2 SOLUTION



$$\begin{aligned} P(\text{passing}) &= 0.2 + (0.8 \times 0.3) + (0.8 \times 0.7 \times 0.4) \\ &= 0.2 + 0.24 + 0.224 \\ &= 0.664 \end{aligned}$$

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PROBLEM 3

A box contains 10 balls of the following types:

3 are red and dotted 2 are gray and dotted
1 is red and striped 4 are gray and striped

If you randomly select one ball, what is the probability that the ball is

Color / Stripe	dotted	striped
Red	3	1
gray	2	4

(a) dotted?
 $5/10$

(b) dotted, given that it is red?
 $3/4$

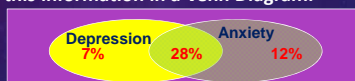
(c) dotted or red?
 $6/10$

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PROBLEM 4

Among the patients at a mental health clinic, 35% suffer from depression and 40% suffer from anxiety. A total of 28% of the patients suffer from both conditions.

(a) Display this information in a Venn Diagram.



Use your Venn Diagram to determine what percent of the patients at this clinic

- (a) ...suffer from depression but not anxiety. 7%
 (b) ...suffer from neither depression nor anxiety. 53%

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PROBLEM 5

Two socks are selected at random and removed in succession (without replacement) from a drawer containing 6 brown socks and 4 blue socks. Let Y represent the number of brown socks selected. Give the probability distribution for Y :

$$Y = \begin{matrix} 0 & 1 & 2 \end{matrix}$$

$$\text{Probability} = \begin{matrix} 12/90 & 48/90 & 30/90 \end{matrix}$$

Support your probabilities by a well labeled tree diagram below:



PROBLEM 6

The probability that a particular type of smoke detector will function properly and sound an alarm in the presence of smoke is 0.9. If there are two of these alarms in my home, what is the probability that at least one works in the presence of smoke?

$$\begin{aligned} P(\text{at least one}) &= 1 - P(\text{none}) \\ &= 1 - (0.1) \times (0.1) \\ &= 1 - 0.01 \\ &= 0.99 \end{aligned}$$

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PROBLEM 7

A manufacturer of airplane parts knows from past experience that the probability is 0.8 that an order will be ready for shipment on time, the probability is 0.6 that an order will be delivered on time, and the probability is 0.5 that an order will be ready for shipment *and* will be delivered on time.

Given: $P(R) = 0.8$ $P(D) = 0.6$ $P(R \text{ and } D) = 0.5$

(a) Find the probability that an order will be delivered on time, given that it is ready for shipment on time.

$$P(D|R) = P(D \text{ and } R) / P(R) = 0.5 / 0.8 = .625$$

(b) Find the probability that a randomly selected order will be ready for shipment on time or will be delivered on time.

$$P(R \text{ or } D) = P(R) + P(D) - P(R \text{ and } D) = 0.8 + 0.6 - 0.5 = 0.9$$

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PROBLEM 8

A box contains 20 fuses, 17 good and 3 defective. Two fuses are drawn from the box *with replacement*.

- (a) What is the probability that both fuses are defective?

$$(3/20) \times (3/20) = 9/400 = 2.25\%$$

- (b) What is the probability that one fuse is good and one is defective?

$$(17/20) \times (3/20) = 51/400 = 12.75\%$$

$$(3/20) \times (17/20) = 51/400 = 12.75\%$$

$$25.5\%$$

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PROBLEM 9

A recent survey asked 100 randomly selected adult Americans if they thought that women should be allowed to go into combat situations. Here are the results:

Gender	Yes	No
Male	32	18
Female	8	42

- (a) Find the probability of a "Yes" answer, given that the person was a female.

$$P(Y | F) = 8 / 50 = 0.16$$

- (b) Find the probability that the respondent was a male, given that the response was a "No."

$$P(M | N) = 18 / 60 = 0.30$$

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PROBLEM 10

Toss two balanced coins. Let A = head on the first toss, and let B = both tosses have the same outcome. Are events A and B independent? Explain your reasoning clearly.

Yes they are independent.

$$P(A) = 1/2 \quad P(B) = 1/2$$

$$P(A \text{ and } B) = P(A) \cdot P(B) \\ = 0.5 \cdot 0.5 \\ = 0.25$$

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PROBLEM 11

Parking for students at Central High School is very limited, and those who arrive late have to park illegally and take their chances at getting a ticket. Joey has determined that the probability that he has to park illegally and that he gets a parking ticket is 0.07. He recorded data last year and found that because of his perpetual tardiness, the probability that he will have to park illegally is 0.25. Suppose that Joey arrived late once again this morning and had to park in a no-parking zone. Can you find the probability that Joey will get a parking ticket? If so, do it. If you need additional information to find the probability, explain what is needed

If T = Joey gets a parking ticket and X = Joey parked illegally then,

$$P(T | X) = P(T \text{ and } X) / P(X) = 0.07 / 0.25 = 0.28$$

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PROBLEM 12

Two cards are dealt, one after the other, from a shuffled 52-card deck. Why is it wrong to say that the probability of getting two red cards is $(1/2)(1/2) = 1/4$? What is the correct probability of this event?

It is wrong because the first card was not replaced and that change the probability of the second draw.

$$\begin{aligned} P(\text{both cards are red}) &= \\ &= P(1^{\text{st}} \text{ card red}) * P(2^{\text{nd}} \text{ card is red} \mid 1^{\text{st}} \text{ card was red}) \\ &= (26/52) * (25/51) \\ &= 0.2451 \end{aligned}$$